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SIES College of Arts, Science and Commerce

(Autonomous)

Affiliated to Mumbai University

Revised Syllabus under Autonomy - June 2023

Program: B.Sc. Mathematics

Semesters V and VI (TYBSc)

Choice Based Credit System (CBCS)

with effect from the academic year 2023-24

CONTENTS

1. Preamble

Mathematics has been fundamental to the development of science and technology. In recent decades, the extent of application of Mathematics to real world problems has increased by leaps and bounds. It is imperative that the content of the undergraduate syllabi of Mathematics should support other branches of science such as Physics, Chemistry, Statistics, Computer Science, Biotechnology. This syllabus of S.Y.B.Sc. Mathematics has been designed to provide learners sufficient knowledge and skills enabling them to undertake further studies in mathematics and its allied areas. There are 6 theory and 2 practical courses spread over semesters III and IV covering the core areas of Mathematics such as Linear Algebra, Real Analysis, Multivariable Calculus, Differential Equations and Numerical Methods.

2. Learning Objectives

- ➢ To develop critical thinking, reasoning and logical skills of the learners
- ➢ To improve learners' analytical and problem solving skills
- ➢ To take the learners from simple to difficult and from concrete to abstract
- ➢ To equip learners with a deeper understanding of abstract theory and concepts
- ➢ To improve learners' capacity to communicate mathematical/logical ideas in writing.

3. Programme Outcomes

SIES has integrated the Learning Outcome Based Curriculum Framework in the syllabi of all the programmes since the academic year 2021-22. Upon completing the B.Sc. Mathematics Programme, the students are expected to develop the following abilities and skills:

I. **Solving Complex Problems:**
Applying the knowledge of various courses learned under a program with an ability to break down complex problems into simple components, by designing processes required for problem solving.

II. **Critical Thinking and reasoning ability:**
Exhibits ability to understand abstract concepts,analyze, and apply them in problem solving. Ability to formulate and develop logical arguments. developing the ability to think with different perspectives and ideas. (Skills necessary for progression to higher education and research.)

III. **Research Aptitude:**
Acquiring the ability to explore and gain knowledge in independent ways through reading assignments, problem solving assignments, projects, seminars, presentations.

IV. **Information and Digital literacy**:
Equip to select, apply appropriate tools and techniques, resources through electronic media for the purpose of visualizing mathematical objects, geometrical interpretations, coding, and analyzing data.

V. **Sound Disciplinary knowledge:**
Demonstrate comprehensive knowledge and understanding of the fundamental concepts and theories of mathematics; apply them to interdisciplinary areas of study.

VI. **Communicating Mathematical Ideas:**
Organize and deliver mathematical ideas through effective written, verbal, graphical/virtual communications.

4. Course structure with minimum credits and Lectures/ Week

SEMESTER V				
Multivariable Integral Calculus				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT51	I	Multiple Integrals	2.5	3
	II	Line Integrals		
	III	Surface Integrals		
Group Theory				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT52	I	Groups and Subgroups	2.5	3
	II	Normal subgroups, Direct products and Cayley's Theorem		
	III	Cyclic groups and cyclic subgroups		
Topology of Metric Spaces				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT53	I	Metric spaces	2.5	3
	II	Complete metric spaces		
	III	Compact Spaces		
Number Theory and Its applications I				
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT54	I	Congruences and Factorization	2.5	3
	II	Diophantine equations and their solutions		
	III	Primitive Roots and Cryptography		
PRACTICALS				
SIUSMATP5A	Practicals based on Courses **SIUSMAT51, SIUSMAT52**		3	6

SIUSMATP5B	Practicals based on Courses **SIUSMAT53, SIUSMAT54**		3	6
		SEMESTER VI		
		Basic complex analysis		
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT61	I	Introduction to Complex Analysis	2.5	3
	II	Cauchy Integral Formula		
	III	Complex power series, Laurent series and isolated singularities		
		RING THEORY		
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT62	I	Rings	2.5	3
	II	Ideals and special rings		
	III	Factorization		

		Topology of Metric Spaces and Real Analysis		
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT63	I	Sequences and series of functions	2.5	3
	II	Continuous functions on Metric spaces		
	III	Connected sets		
		Number Theory and Its applications II		
Course Code	UNIT	TOPICS	Credits	L/Week
SIUSMAT64	I	Quadratic Reciprocity	2.5	3
	II	Continued Fractions		
	III	Pell's equation, Arithmetic functions & special functions		

PRACTICALS			
SIUSMATP6A	Practicals based on Courses **SIUSMAT61, SIUSMAT62**	3	6
SIUSMATP6B	Practicals based on Courses **SIUSMAT63, SIUSMAT64**	3	6

5. Consolidated Syllabus for semester V & VI with Course Outcomes

SEMESTER V

Course: Multivariable Integral Calculus
Course Code: SIUSMAT51

Expected Course Outcomes:

On completion of this course, the students are expected to

1. State the definitions and prove results based on concepts of multiple, line and surface integration
2. Apply various definitions learnt to identify and plot bounded regions, compute double and triple integrals, line and surface integrals.
3. Test the validity of mathematical statements and converses based upon the gained knowledge.

Unit I - Multiple Integrals (15 Lectures)

(i)Definition of double (resp: triple) integral of a bounded function on a rectangle (resp: box). Geometric interpretation as area and volume. Fubini's Theorem over rectangles and on any closed bounded sets, Iterated Integrals.

(ii)Integrability of the sums, scalar multiples, products, and (under suitable conditions) quotients of integrable functions. Formulae for the integrals of sums and scalar multiples of integrable functions.

(iii) Integrability of continuous functions. More generally, integrability of functions with a set of discontinuities of measure zero. (concept and examples only)

(iv)Domain additivity of the integral. Integrability and the integral over arbitrary bounded domains. Change of variables formula (Statement only).

(v)Polar, cylindrical and spherical coordinates, and integration using these coordinates. Differentiation under the integral sign using Leibnitz Rule. Applications to finding the center of gravity and moments of inertia.

Unit II: Line Integrals (15 Lectures)

(i) Review of Scalar and Vector fields on $\mathbb{R}^n$, Vector Differential Operators, Gradient, Curl, Divergence.

(ii) Paths (parametrized curves) in $\mathbb{R}^n$(emphasis on $\mathbb{R}^2$and $\mathbb{R}^3$), Smooth and piecewise smooth paths. Closed paths. Equivalence and orientation preserving equivalence of paths.

(iii) Definition of the line integral of a scalar and a vector field over a piecewise smooth path. Basic properties of line integrals including linearity, path additivity and behavior under a change of parameters. Examples.

(iv) Line integrals of the gradient vector field, Fundamental Theorem of Calculus for Line Integrals, Necessary and sufficient conditions for a vector field to be conservative.

(v) Green's Theorem (proof in the case of rectangular domains). Applications to evaluation of line integrals.

Unit III: Surface Integrals (15 Lectures)

(i) Parameterized surfaces. Smoothly equivalent parameterizations. Area of such surfaces.

(ii) Definition of surface integrals of scalar valued functions as well as of vector fields defined on a surface. Examples.

(iii) Curl and divergence of a vector field. Elementary identities involving gradient, curl and divergence.

(iv) Stokes' Theorem (proof assuming the general from of Greens Theorem). Examples.

(v) Gauss Divergence Theorem (proof only in the case of cubical domains). Examples. References

1. Apostol Tom, *Calculus Vol. 2*, Second Ed., John Wiley, New York, 1969
2. James Stewart , *Calculus: early transcendentals*, Cengage SA, 8th edition
3. J. E. Marsden and A. J. Tromba, *Vector Calculus*, Freeman, Fifth edition, 2003
4. G.B. Thomas and R.L Finney, *Calculus and Analytic Geometry,* Ninth Ed. (ISE Reprint), Addison Wesley, Reading Mass, 1998.

Other References :

1. R. Courant and F. John, *Introduction to Calculus and Analysis*, Vol.2, Springer Verlag, New York, 1989.
2. M.H. Protter and C. B. Morrey Jr., *Intermediate Calculus*, Second Ed., Springer Verlag, New York, 1995.
3. Sudhir R. Ghorpade and Balmohan Limaye, A Course in Multivariable Calculus and Analysis
4. Springer International Edition, 2009.

Online Open Resources:

1. Paul's online notes: Calculus III http://tutorial-math.wip.lamar.edu/Classes/CalcIII/CalcIII.aspx
2. OP Calculus volume 3
3. https://openstax.org/details/books/calculus-volume-3

Course: Group Theory

Course Code: SIUSMAT52

On completion of this course, students are expected to

1. Express understanding of the fundamental concepts of group theory, including groups, subgroups, cosets, homomorphisms and the properties of group operations. Write proofs of important theorems in group theory, such as Lagrange's theorem, Cauchy's theorem and the classification of finite abelian groups.
2. Apply learnt knowledge in constructing proofs, understanding and solving problems related to subgroups, normal subgroups, and cyclic groups.
3. Be able to analyze and solve a variety of examples of groups, such as permutation groups, matrix groups, cyclic groups, dihedral groups, and symmetric groups.

Unit I. Groups and Subgroups (15 Lectures)

(i) Definition and elementary properties of a group. Order of a group. Subgroups. Criterion for a subset to be a subgroup. Abelian groups. Center of a group. Homomorphisms and isomorphisms.

(2) Examples of groups including Z, Q, R, C, Klein 4-group, symmetric and alternating groups,S1(= the unit circle in C), GLn(R), SLn(R), On (= the group of n × n nonsingular upper triangular matrices), Bn (= the group of n × n nonsingular upper triangular matrices),and groups of symmetries of plane figures.

(3) Order of an element. Subgroup generated by a subset of the group.

Unit II. Normal subgroups, Direct products and Cayley's Theorem (15L)

(i) Cosets of a subgroup in a group. Lagrange's Theorem. Normal subgroups. Alternating group An. Listing normal subgroups of A4, S3. Quotient (or Factor) groups. Fundamental Theorem of homomorphisms of groups.

(ii) External direct products of groups. Examples. Relation with internal products such as HK of subgroups H, K of a group.

(iii) Cayley's Theorem for finite groups

Unit III: Cyclic groups and cyclic subgroups (15L)

(i) Examples of cyclic groups such as Z and the group μn of the n−th roots of unity. Properties of cyclic groups and cyclic subgroups.

(ii) Finite cyclic groups, infinite cyclic groups and their generators. Properties of generators.
(iii) The group Z/nZ of residue classes (mod n). Characterization of cyclic groups (as being isomorphic to Z or Z/nZ for some n $\in$ N).

Recommended Books.

1. Israel Herstein; Topics in Algebra; Second edition, Wiley Eastern Limited.
2. P. B. Bhattacharya, S.K. Jain, S. Nagpaul; Abstract Algebra; Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalkrishnan, University Algebra, Wiley Eastern Limited.
4. Michael Artin; Algebra; Prentice Hall of India, New Delhi.
5. John Fraleigh; A first course in Abstract Algebra; third edition, Narosa, New Delhi.
6. Joseph Gallian; Contemporary Abstract Algebra; Narosa, New Delhi.

Additional Reference Books

1. Thomas Hungerford; Algebra; Springer.
2. David Dummit; Richard Foote; Abstract Algebra; John Wiley & Sons, Inc.
3. L.S. Luthar and I.B.S. Passi, Algebra. Vol. I and II, Narosa.

Course: Topology of Metric Spaces

Course Code: SIUSMAT53

On completion of this course, students are expected to

1. State the definitions and prove the theorems of open and closed ball, open and closed set, limit point, interior, closure, boundary point, distance between two sets, diameter of a set, equivalent metrics, subspaces, cauchy sequences, complete metric spaces, compact metric spaces.
2. Prove the statements and solve problems based on metric, open and closed sets, limits, interior, closure, equivalent metrics, Cauchy sequences, complete metric spaces, compact sets.
3. Identify whether sets are open, closed, complete, compact

Unit I: Metric spaces (15 Lectures)

Definition, examples of metric spaces $\mathbb{R}$; $\mathbb{R}^2$,Euclidean space $\mathbb{R}^n$ with its Euclidean, sup and sum metric, $\mathbb{C}$ (complex numbers), the spaces l^1 and l^2 of sequences and the space C[a; b], of real valued continuous functions on [a; b]. Discrete metric space.
Distance metric induced by the norm, translation invariance of the metric induced by the norm. Metric subspaces, Product of two metric spaces. Open balls and open set in a metric space, examples of open sets in various metric spaces. Hausdorff property. Interior of a set. Properties of open sets. Structure of an open set in $\mathbb{R}$. Equivalent metrics.

Distance of a point from a set, between sets, diameter of a set in a metric space and bounded sets. Closed ball in a metric space, Closed sets definition, examples. Limit point of a set, isolated point, a closed set contains all its limit points, Closure of a set and boundary of a set.

Unit II: Complete metric spaces (15 Lectures)

Sequences in a metric space, Convergent sequence in metric space, Cauchy sequence in a metric space, subsequences, examples of convergent and Cauchy sequence in finite metric spaces, $\mathbb{R}^n$ with different metrics.

Characterization of limit points and closure points in terms of sequences, Definition and examples of relative openness/closeness in subspaces. Dense subsets in a metric space and Separability. Definition of complete metric spaces, Examples of complete metric spaces, Completeness property in subspaces, Nested Interval theorem in $\mathbb{R}$, Cantor's Intersection Theorem, Applications of Cantor's Intersection Theorem:

(i) The set of real Numbers is uncountable.

(ii) Density of rational Numbers(Between any two real numbers there exists a rational number)

(iii) Intermediate Value theorem.

Unit III: Compact sets (15 lectures)

Definition of compact metric space using open cover, examples of compact sets in different metric spaces $\mathbb{R}$; $\mathbb{R}^2$;$\mathbb{R}^n$, Properties of compact sets: A compact set is closed and bounded, (Converse is not true). Every infinite bounded subset of compact metric space has a limit point. A closed subset of a compact set is compact. Union and Intersection of Compact sets. Equivalent statements for compact sets in R:

(i) Sequentially compactness property.

(ii) Heine-Borel property.

(iii) Closed and boundedness property.
(iv) Bolzano-Weierstrass property.

Reference books:

1. S. Kumaresan, Topology of Metric spaces.

2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996. 3.
Expository articles of MTTS programme

4. P. K. Jain, Khalil Ahmad. Metric Spaces. Narosa. New Delhi, 1996.

Other references :

1. W. Rudin, Principles of Mathematical Analysis.

2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

3. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

4. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

5. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi

6. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.

7. Sutherland. Topology.

Course: Number Theory and its applications I

Course Code: SIUSMAT54

On completion of this course, students are expected to

1. State the definitions and prove the results based on concepts of congruences and factorisation, linear and nonlinear Diophantine Equations, primitive roots
2. Apply various definitions and theorems to solve problems based on congruences and factorisation, Diophantine equations, different types of cryptosystems
3. Test the validity of mathematical statements and converses based upon the gained knowledge.

Unit I. Congruences and Factorization (15 Lectures)

Review of Divisibility, Primes and The fundamental theorem of Arithmetic. Congruences : Definition and elementary properties, Complete residue system modulo m, Reduced residue system modulo m, Euler's function and its properties, Fermat's little Theorem, Euler's generalization of Fermat's little Theorem, Wilson's theorem, Linear congruence, The Chinese remainder Theorem, Congruences of Higher degree, The Fermat-Kraitchik Factorization Method.

Unit II. Diophantine equations and their solutions (15 Lectures)

The linear equations $ax + by = c$. The equations $x^2 + y^2 = p$; where p is a prime. The equation $x^2 + y^2 = z^2$, Pythagorean triples, primitive solutions. The equations $x^4 + y^4 = z^2$ and $x^4 + y^4 = z^4$ have no solutions (x, y, z) with $xyz \neq 0$. Every positive integer n can be expressed as sum of squares of four integers. Assorted examples : section 5.4 of Number theory by Niven-Zuckermann-Montgomery.

Unit III. Primitive Roots and Cryptography (15 Lectures)

Order of an integer and Primitive Roots. Basic notions such as encryption (enciphering) and decryption (deciphering), Cryptosystems, symmetric key cryptography, Simple examples such as Shift cipher, Affine cipher, Hill's cipher, Vigénere cipher . Concept of Public Key Cryptosystem; EIGamal cryptosystem, RSA Algorithm. An application of Primitive Roots to Cryptography.

References:

1. Niven, H. Zuckerman and H. Montogomery, An Introduction to the Theory of Numbers, John Wiley & Sons. Inc.

2. David M. Burton, An Introduction to the Theory of Numbers. Tata McGraw Hill Edition.

3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. Neville Robins. Beginning Number Theory. Narosa Publications.

5. S.D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House.

6. N. Koblitz. A course in Number theory and Cryptography, Springer. 7. M. Artin, Algebra. Prentice Hall.

8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

9. William Stalling. Cryptology and network security.

Course SIUSMATP5A: Practicals (Based on SIUSMAT51 and SIUSMAT52)

Practicals based on SIUSMAT51

1. Evaluation of double and triple integrals.

2. Change of variables in double and triple integrals and applications 3. Line integrals of scalar and vector fields

4. Greens theorem, conservative field and applications

5. Evaluation of surface integrals

6. Stokes and Gauss divergence theorem

7. Miscellaneous theory questions on units 1, 2 and 3.

Practicals based on SIUSMAT52

1. Examples of groups and groups of symmetries of equilateral triangle, square and rectangle.
2. Examples of determining centers of different groups. Examples of subgroups of various groups and orders of elements in a group.
3. Left and right cosets of a group and Lagrange's theorem.
4. Normal subgroups and quotient groups. Direct products of groups.
5. Finite cyclic groups and their generators
6. Infinite cyclic groups and their properties.
7. Miscellaneous Theory Questions

Course SIUSMATP5B: Practicals (Based on SIUSMAT53 and SIUSMAT54)

Practicals based on SIUSMAT53

1. Examples of Metric Spaces, Normed Linear Spaces,
2. Sketching of Open Balls in $\mathbb{R}^2$, Open and Closed sets, Equivalent Metrics
3. Subspaces, Interior points, Limit Points, Dense Sets and Separability, Diameter of a set, Closure.
4. Sequences , Bounded , Convergent Sequences.
5. Cauchy sequences and Complete Metric Spaces.
6. Examples of Compact Sets
7. Miscellaneous Theory Questions based on units 1, 2, 3

Practicals based on SIUSMAT54

Use of non-programmable scientific calculators is allowed in practicals of this paper. 1. Congruences.

2. Linear congruences and congruences of Higher degree.
3. Linear diophantine equation.
4. Pythagorean triples and sum of squares.
5. Cryptosystems (Private Key).
6. Cryptosystems (Public Key) and primitive roots.

7. Miscellaneous theoretical questions based on units 1, 2, and 3.

SEMESTER VI

Course: Basic Complex Analysis **Course Code: SIUSMAT61**

Course Outcomes

On completion of this course, students are expected to

1. State the definitions and establish theoretical results in basic complex analysis, define and identify analytic functions, singularities, residues, plot regions in complex plane
2. Demonstrate the ideas of complex differentiation, integration and residues for solving related problems, representing a function as a Taylor's / Laurent's series.
3. Test the validity of mathematical statements and converses based upon the gained knowledge, identify the type of singularity, type of region.

Unit I: Introduction to Complex Analysis (15 Lectures)

(i) Review of complex numbers: Complex plane, polar coordinates, exponential map, powers and roots of complex numbers, De Moivre's formula, C as a metric space, bounded and unbounded sets, point at infinity-extended complex plane, sketching of sets in complex plane.

(ii) Limit at a point, theorems on limits, convergence of sequences of complex numbers and results using properties of real sequences. Functions $f : C \rightarrow C$, real and imaginary part of functions, continuity at a point and algebra of continuous functions.

(iii) Derivative of $f : C \rightarrow C$, comparison between differentiability in real and complex sense, Cauchy-Riemann equations, sufficient conditions for differentiability, Algebra of analytic function, chain rule.

(iv) Theorem: If $f'(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D. Harmonic functions and harmonic conjugate.

Unit II: Cauchy Integral Formula (15 Lectures)

(i) Evaluation and basic properties of complex line integral.

(ii)Cauchy theorem, Cauchy-Goursat theorem

(iii) Cauchy integral formula, Cauchy estimates

(iv) Liouville's theorem and applications(proof of Fundamental Theorem of Algebra)

(v)Taylor's theorem for analytic function, Zeros of an analytic function

(vi) Exponential function, its properties, trigonometric function, hyperbolic functions.

(vii) Fractional Linear Transformations: definition and examples.

Unit III: Complex power series, Laurent series and isolated singularities. (15 Lectures)

(i) Power series of complex numbers and related results following from Unit I, radius of convergence, disc of convergence, uniqueness of series representation, examples.
(ii) Definition of Laurent series , Definition of isolated singularity, statement (without proof) of existence of Laurent series expansion in neighborhood of an isolated singularity, type of isolated singularities viz. Removable singularity, pole and essential singularity, defined using Laurent series expansion, examples.
(iii) Statement of Residue theorem and calculation of residue.

Reference:

1. J.W. Brown and R.V. Churchill, Complex analysis and Applications

Other References:

1. Robert E. Greene and Steven G. Krantz, Function theory of one complex variable 2. T.W. Gamelin, Complex analysis

3. Lars Ahlfors , Complex Analysis, Third Edition. McGrawHill Education.

Course: Ring Theory Course Code: SIUSMAT62

On completion of this course, students are expected to

1. Express understanding of the basic properties and structures of rings, including the definition of a ring, subrings, ideals, homomorphisms, and isomorphisms, state the important results and write theoretical proofs.
2. Apply learnt knowledge to solve examples of rings, such as the integers, polynomial rings, matrix rings, quotient rings, and Euclidean domains, applying properties of rings, and solving problems related to ideals, quotient rings, and ring homomorphisms.
3. Analyze and interpret algebraic expressions within the context of rings, including working with polynomial operations, factorization, and solving equations.

Unit I. Rings (15 Lectures)

(1) Definition and elementary properties of rings (where the definition should include the existence of unity), commutative rings, integral domains and fields. Examples, including Z, Q, R, Z/nZ, C, Mn(R), Z[i], Z[√2], Z[√−5], Z[X], R[X], C[X],(Z/nZ)[X].
(2) Units in a ring. The multiplicative group of units in a ring R [and, in particular, the multiplicative group F ∗ of nonzero elements of a field F]. Description of the units in Z/nZ. Results such as: A finite integral domain is a field. Z/pZ, where p is a prime, as an example of a finite field.
(3) Characteristic of a ring. Examples. Elementary facts such as: the characteristic of an integral domain is either 0 or a prime number.

(Note: From here on all rings are assumed to be commutative with unity).

Unit II. Ideals and special rings(15L)

(1) Ideals in a ring. Sums and products of ideals. Quotient rings. Examples. Prime ideals and maximal ideals. Characterization of prime ideals and maximal ideals in a commutative ring in terms of their quotient rings. Description of the ideals and the prime ideals in Z, R[X] and C[X].

(2) Homomorphisms and isomorphism of rings. Kernel and the image of a homomorphism.Fundamental Theorem of homomorphism of a ring.

(3) Construction of the quotient field of an integral domain (Emphasis on Z, Q). A field contains a subfield isomorphic to Z/pZ or Q.

(4) Notions of euclidean domain (ED), principal ideal domain (PID). Examples such as Z, Z[i],and polynomial rings. Relation between these two notions (ED ⇒ PID).

Unit III. Factorization (15L)

(1) Divisibility in a ring. Irreducible and prime elements with examples.

(2) Division algorithm in F[X] (where F is a field). Monic polynomials, greatest common divisor of f(x), g(x) ∈ F[X] (not both 0). Theorem: Given f(x) and g(x) 6= 0, in F[X] then their greatest common divisor d(x) ∈ F[X] exists; moreover, d(x) = a(x)f(x) + b(x)g(x) for some a(x), b(x) ∈ F[X]. Relatively prime polynomials in F[X], irreducible polynomials in F[X]. Examples of irreducible polynomials in (Z/pZ)[X] (p prime), Eisenstein Criterion (without proof).

(3) Notion of unique factorization domain (UFD). Elementary properties. Example of a non-UFD is Z[√−5] (without proof). Theorem (without proof). Relation between the three notions (ED ⇒ PID ⇒ UFD). Examples such as Z[X] of UFD that are not PID.
Theorem (without proof): If R is a UFD, then R[X] is a UFD.

Reference Books

1. Israel Herstein; Topics in Algebra; Second edition, Wiley Eastern Limited, Second edition.
2. P. B. Bhattacharya, S. K. Jain, and S. R. Nagpaul; Abstract Algebra; Second edition, Foundation Books, New Delhi, 1995.
3. N. S. Gopalakrishnan; University Algebra; Wiley Eastern Limited.
4. Michael Artin; Algebra; Prentice Hall of India, New Delhi.
5. John. B. Fraleigh; A First course in Abstract Algebra; Third edition, Narosa, New Delhi.
6. Joseph A. Gallian; Contemporary Abstract Algebra; Narosa, New Delhi.

Additional Reference Books:

1. Sukumar Das Adhikari; An Introduction to Commutative Algebra and Number theory; Narosa Publishing House.
2. Thomas W. Hungerford; Algebra; Springer.
3. David S. Dummit, Richard Foote; Abstract Algebra; John Wiley & Sons, Inc.
4. L.S. Luthar and I.B.S. Passi, Algebra. Vol. I and II, Narosa.
5. U. M. Swamy, A. V. S. N. Murthy; Algebra Abstract and Modern; Pearson.
6. Charles Lanski; Concepts Abstract Algebra; American Mathematical Society.
7. Sen, Ghosh and Mukhopadhyay; Topics in Abstract Algebra; Universities press.

Course : Topology of Metric Spaces and Real Analysis

Course Code: SIUSMAT63

On completion of this course, students are expected to

1. State the definitions and prove the theorems of sequence and series of functions, continuity and uniform continuity, separated sets, connected sets.
2. Examine pointwise and uniform convergence of sequence and series of functions, continuity and uniform continuity of functions, connectedness of sets,

Unit I : Sequence and series of functions (15 lectures)

Sequence of functions - pointwise and uniform convergence of sequences of real- valued functions, examples. Uniform convergence implies pointwise convergence, example to show converse is not true, series of functions, convergence of series of functions, Weierstrass M-test(Statement only). Examples. Properties of uniform convergence: Continuity of the uniform limit of a sequence of continuous function, conditions under which integral and the derivative of sequence of functions converge to the integral and derivative of uniform limit on a closed and bounded interval. (Statements only) Examples. Consequences of these properties for a series of functions, term by term differentiation and integration. (Statements only) Power series in $\mathbb{R}$ centered at origin and at some point in $\mathbb{R}$, radius of convergence, region (interval) of convergence, uniform convergence, term by-term differentiation and integration of power series, Examples. Uniqueness of series representation, functions represented by power series, classical functions defined by power series such as exponential, cosine and sine functions, the basic properties of these functions.

Unit II: Continuous functions on metric spaces (15 Lectures)

Continuous functions on metric spaces Epsilon-delta definition of continuity at a point of a function from one metric space to another. Characterization of continuity at a point in terms of sequences, open sets and closed sets and examples, Algebra of continuous real valued functions on a metric space. Continuity of composite continuous function. Continuous image of compact set is compact, Uniform continuity in a metric space, definition and examples (emphasis on $\mathbb{R}$).Results such as: every continuous functions from a compact metric space is uniformly continuous. Contraction mapping and fixed point theorem, Applications.

Unit III: Connected sets (15 Lectures)

Separated sets- Definition and examples, disconnected sets, disconnected and connected metric spaces, Connected subsets of a metric space, Connected subsets of $\mathbb{R}$. A subset of $\mathbb{R}$ is connected if and only if it is an interval. A continuous image of a connected set is connected. Characterization of a connected space, viz. a metric space is connected if and only if every continuous function from X to $\{1, -1\}$ is a constant function. Path connectedness in $\mathbb{R}^n$, definition and examples. A path connected subset of $\mathbb{R}^n$ is connected, convex sets are path connected.

Connected components. An example of a connected subset of $\mathbb{R}^n$ which is not path connected.

References for Units I, II, III:

1. S. Kumaresan, Topology of Metric spaces.

2. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

3. Robert Bartle and Donald R. Sherbert, Introduction to Real Analysis, Second Edition, John Wiley and Sons.

4. Ajit Kumar, S. Kumaresan, Introduction to Real Analysis

5. R.R. Goldberg, Methods of Real Analysis, Oxford and International Book House (IBH) Publishers, New Delhi.

6. P.K.Jain. K. Ahmed. Metric Spaces. Narosa, New Delhi, 1996.

Other references :

1. W. Rudin, Principles of Mathematical Analysis.

2. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974

3. E. T. Copson. Metric Spaces. Universal Book Stall, New Delhi, 1996.

4. R. R. Goldberg Methods of Real Analysis, Oxford and IBH Pub. Co., New Delhi 1970.

5. W. Rudin. Principles of Mathematical Analysis, Third Ed, McGraw-Hill, Auckland, 1976.

6. D. Somasundaram, B. Choudhary. A first Course in Mathematical Analysis. Narosa, New Delhi

7. G.F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hi, New York, 1963.

Course: Number Theory and its applications II

Course Code: SIUSMAT64

On completion of this course, students are expected to

1. State the definitions and prove the results based on concepts of quadratic reciprocity, finite and infinite continued fractions, Pell's equation, Number Theoretic functions and special numbers.

2. Apply various definitions and theorems to solve problems based on quadratic using Legendre Symbol and other techniques, approximating irrational numbers using continued fractions, Pell's equation, Number Theoretic functions and special numbers.
3. Test the validity of mathematical statements and converses based upon the gained knowledge.

Unit I. Quadratic Reciprocity (15 Lectures)

Quadratic residues and Legendre Symbol, Gauss Lemma, Theorem on Legendre Symbol $\left(\frac{2}{p}\right)$ and $\left(\frac{3}{p}\right)$, and associated results. If p is an odd prime and a is an odd integer with (a, p) = 1 then $\left(\frac{a}{p}\right) = (-1)^t$ where $t = \sum_{k=1}^{\frac{p-1}{2}} [\frac{ka}{p}]$. Quadratic Reciprocity law.

The Jacobi Symbol and law of reciprocity for Jacobi Symbol. Quadratic Congruences with Composite moduli.

Unit II. Continued Fractions (15 Lectures)

Finite continued fractions. Infinite continued fractions and representation of an irrational num-ber by an infinite simple continued fraction, Rational approximations to irrational numbers and order of convergence, Best possible approximations. Periodic continued fractions.

Unit III. Pell's equation, Arithmetic functions and Special numbers (15 Lectures)

Pell's equation $x^2 - dy^2 = 1$, where d is not a square of an integer. Solutions of Pell's equation (The proofs of convergence theorems to be omitted). Arithmetic functions of number theory: $d(n)$ (or $\tau(n)$); $\sigma(n)$; $\sigma_k(n)$; $\omega(n)$ and their properties, $\mu(n)$ and the $M\ddot{o}bius$ inversion formula. Special numbers: Fermat numbers, Mersenne numbers, Perfect numbers, Amicable numbers, Pseudo primes, Carmichael numbers.

Recommended Books

1. Niven H. Zuckerman and H. Montogomery. An Introduction to the Theory of Numbers. John Wiley & Sons. Inc.

2. David M. Burton. An Introduction to the Theory of Numbers. Tata McGraw-Hill Edition.

3. G. H. Hardy and E.M. Wright. An Introduction to the Theory of Numbers. Low priced edition. The English Language Book Society and Oxford University Press, 1981.

4. Neville Robins. Beginning Number Theory. Narosa Publications.

5. S. D. Adhikari. An introduction to Commutative Algebra and Number Theory. Narosa Publishing House

6. .N. Koblitz. A course in Number theory and Cryptography. Springer. 7. M.

Artin. Algebra. Prentice Hall.

8. K. Ireland, M. Rosen. A classical introduction to Modern Number Theory. Second edition, Springer Verlag.

9. William Stalling. Cryptology and network security.

Course SIUSMATP6A: Practicals (Based on SIUSMAT61 and SIUSMAT62)

Practicals based on SIUSMAT61

1. Limit continuity and derivatives of functions of complex variables

2. Stereographic Projection , Analytic function, finding harmonic conjugate 3. Contour Integral, Cauchy Integral Formula , Mobius transformations 4. Taylor's Theorem , Exponential , Trigonometric, Hyperbolic functions 5. Power Series , Radius of Convergence, Laurent’s Series

6. Finding isolated singularities, removable, pole and essential, Cauchy Residue theorem.

7. Miscellaneous theory questions based on Unit 1, 2 and 3.

Practicals based on SIUSMAT62

1. Examples of rings (commutative and non-commutative), integral domains and fields
2. Units in various rings. Determining characteristics of rings.
3. Prime Ideals and Maximal Ideals, examples on various rings.
4. Euclidean domains and principal ideal domains (examples and non-examples)
5. Examples of irreducible and prime elements.
6. Applications of division algorithm and Eisenstein’s criterion.
7. Miscellaneous Theoretical questions on Unit 1, 2 and 3.

Course SIUSMATP6B: Practicals (Based on SIUSMAT63 and SIUSMAT64)

Practicals based on SIUSMAT63

1. Pointwise and uniform convergence of sequence functions, properties
2. Point wise and uniform convergence of series of functions and properties
3. Continuity in a Metric Spaces
4. Uniform Continuity, Contraction maps, Fixed point theorem
5. Connected Sets , Connected Metric Spaces
6. Path Connectedness, Convex sets, Continuity and Connectedness

7. Miscellaneous Theory Questions based on Unit 1, 2 and 3.

Practicals based on SIUSMAT64

1. Legendre Symbol.
2. Jacobi Symbol and Quadratic congruences with composite moduli.
3. Finite continued fractions.
4. Infinite continued fractions.
5. Pell’s equations and Arithmetic functions of number theory.
6. Special Numbers.
7. Miscellaneous Theoretical questions based on unit 1, 2 and 3.

6. Teaching Pattern for semester V & VI

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical of three periods per week per course (1 lecture/period is of 48 minutes duration).

Note: All topics have to be covered with proof in detail (unless mentioned otherwise) and examples.

7. Scheme of Evaluation for Semesters V & VI

The performance of the learners shall be evaluated in three ways:
(a) Continuous Internal Assessment of 40 marks in each course in each semester. (b) Semester End Examinations of 60 marks in each course at the end of each semester. (c) A Practical exam of 200 marks for all the four courses at the end of each semester.

(a) Internal Assessment in each Course in each semester

Sr No	Evaluation type	Marks
1	One class test	20
2	Presentation/ Project /Assignment	10
3	Viva Voce	10
Total		40

(b) Semester end examination in each course at the end of each semester (60 marks)

Duration – 2 hours .

Question Paper Pattern:- Four questions each of 15 marks.

Question Nos 1, 2 and 3 will be on unit I, II, III respectively.

Question 4 will be based on entire syllabus.

All questions shall be compulsory with not more than 50% internal choice within the questions. Question may be subdivided into sub-questions a, b, c.

(c) Practical Examination of 100 marks in each course at the end of each semester

Practical Course	Sem	Part A	Parts B	Marks out of	Duration	Journal	Viva
SIUSMATP5A	V	Questions based on SIUSMAT51	Questions based on SIUSMAT52	80	3 Hours	10 marks	10 marks
SIUSMATP5B	V	Questions based on SIUSMAT53	Questions based on SIUSMAT54	80	3 Hours	10 marks	10 marks
SIUSMATP6A	VI	Questions based on SIUSMAT61	Questions based on SIUSMAT62	80	3 Hours	10 marks	10 marks
SIUSMATP6B	VI	Questions based on SIUSMAT63	Questions based on SIUSMAT64	80	3 Hours	10 marks	10 marks

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